Choice Set Optimization Under Discrete Choice Models of Group Decisions

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Discrete choice models

Goal

Model human choices

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Given a set of items, produce probability distribution

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Multinomial logit (MNL) model (McFadden, 1974)

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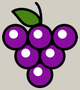
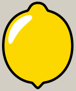
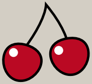
Choice set					
Utility	2	3	2	1	1

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Utility	2	3	2	1	1

$\downarrow$ softmax

Choice prob.	0.18	0.50	0.18	0.07	0.07

$$\Pr(\text{choose } x \text{ from choice set } C) = \frac{\exp(u_x)}{\sum_{y \in C} \exp(u_y)}$$

The choice set influences preferences

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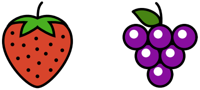
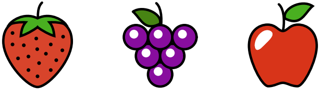
e.g., preference for red fruit:

choice set 1



4 1

choice set 2



2 1 **2**

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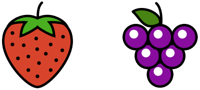

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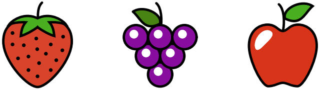

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Not expressible with MNL

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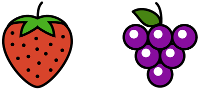

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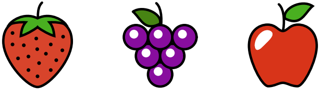
choice set 2



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Context effects are common
(Huber et al., 1982; Simonson & Tversky, 1992; Shafir et al., 1993; Trueblood et al., 2013)

Title breakdown

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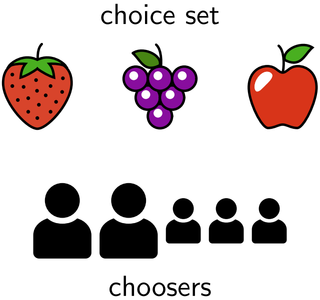

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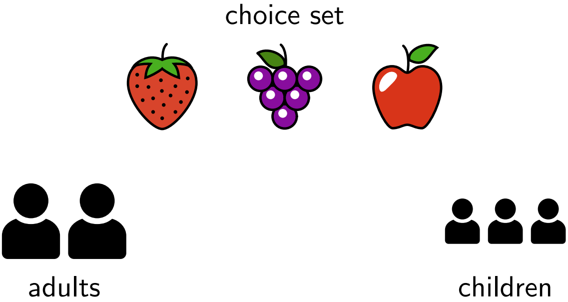

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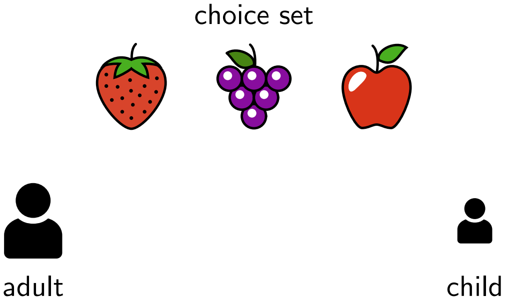

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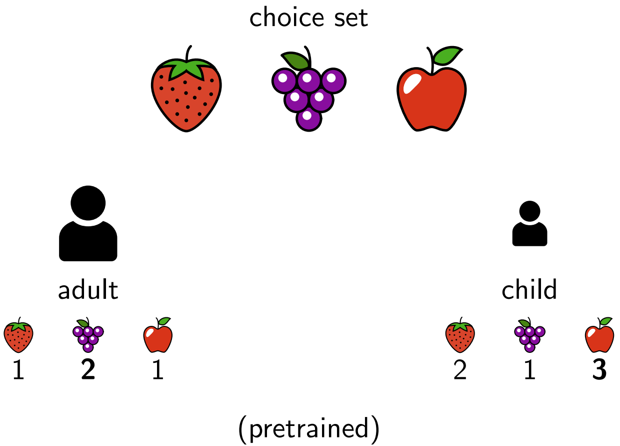

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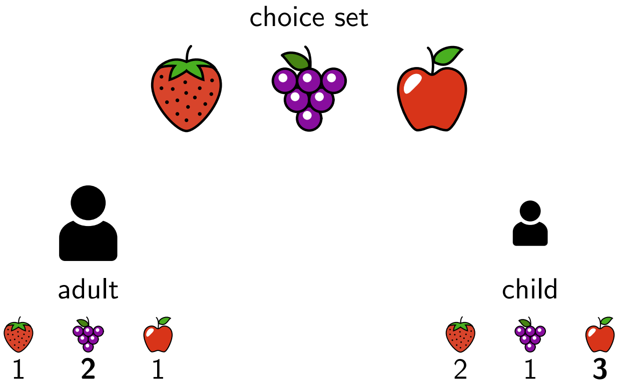

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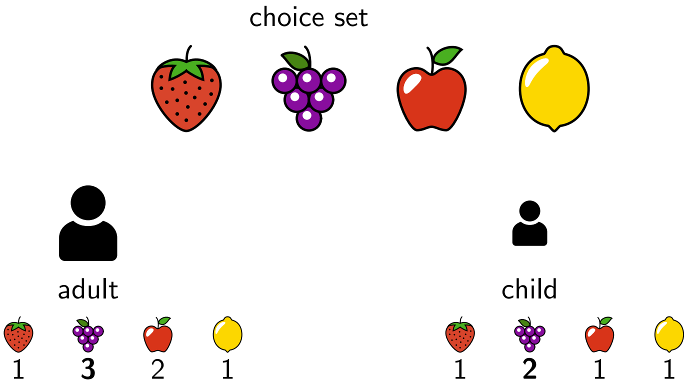

Our contributions

Central algorithmic question

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*See paper

Three models accounting for context effects

- Nested logit (NL) (McFadden, 1978)
- Context-dependent random utility model (CDM) (Seshadri et al., 2019)
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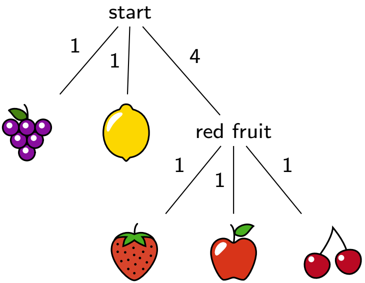

repeated softmax
over node children

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p_{xy}					
		0	-1	0	-1
	0		0	0	0
	-1	0		0	-1
	0	0	0		0
	-1	0	-1	0	

softmax over
pull-adjusted utilities:

$$u_x + \sum_{z \in C} p_{zx}$$

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item	aspects
	{berry, red, sweet}
	{berry, purple, sweet}
	{red, crunchy}
	{citrus, yellow, sour}
	{red, sweet}

utility for each *aspect*

repeatedly choose an aspect,
eliminate items without it

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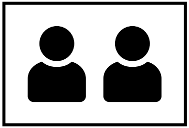
3. Can learn utilities from choice data (SGD on NLL)

Outline

Problem setup

A 

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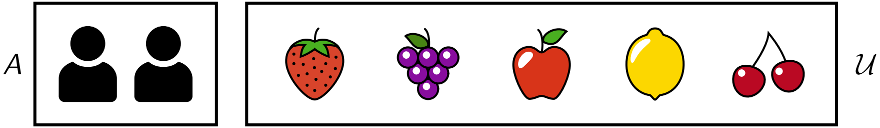

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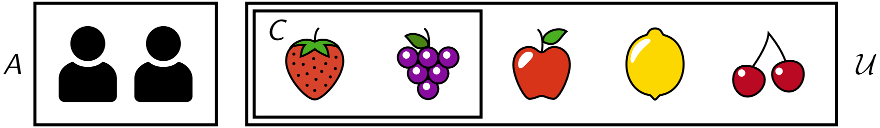

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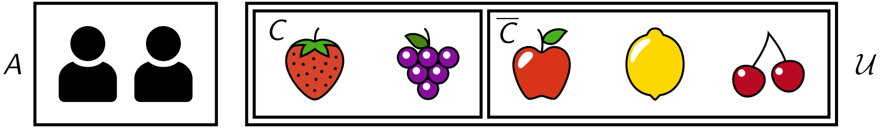

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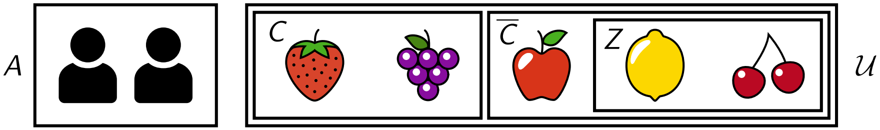

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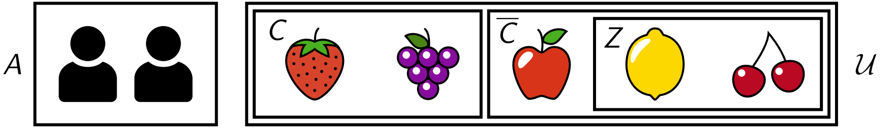

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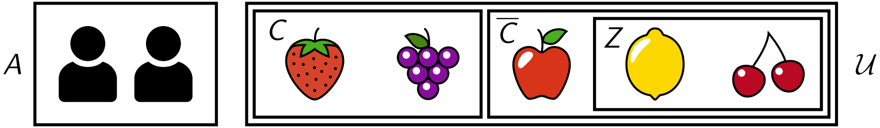

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Choice set optimization

Find $Z \subseteq \overline{C}$ that optimizes some function of $\Pr(a \leftarrow x \mid C \cup Z)$

Three choice set optimization problems

Disagreement induced by Z

$$D(Z) = \sum_{\substack{\{a,b\} \subseteq A \\ x \in C}} \left|\Pr(a \leftarrow x \mid C \cup Z) - \Pr(b \leftarrow x \mid C \cup Z)\right|$$

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Promotion

Find Z that maximizes number of people whose favorite item in C is x^*

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NL, CDM, and EBA $\textsc{Agreement}/\textsc{Disagreement}$ *are NP-hard.*

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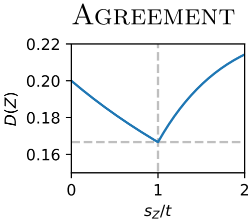
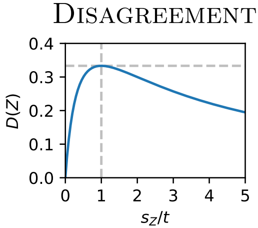
SUBSET SUM reductions





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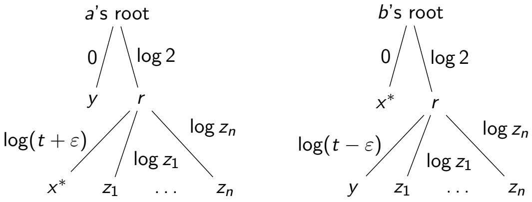

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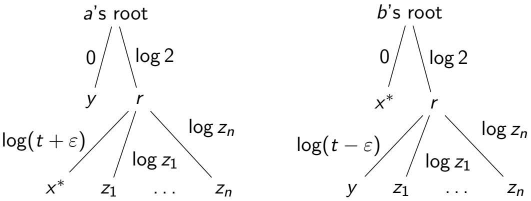

Promotion is "easier" than Agreement

Model restrictions make Promotion easy, but leave Agreement hard.

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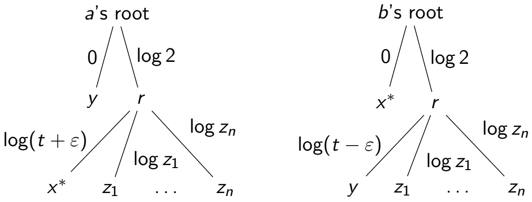

PROMOTION is "easier" than AGREEMENT

Model restrictions make PROMOTION easy, but leave AGREEMENT hard.
e.g., same-tree NL

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Poly-time approximation for small group AGREEMENT

Idea (inspired by SUBSET SUM FPTAS from CLRS)

Discretize possible utility sums of Zs

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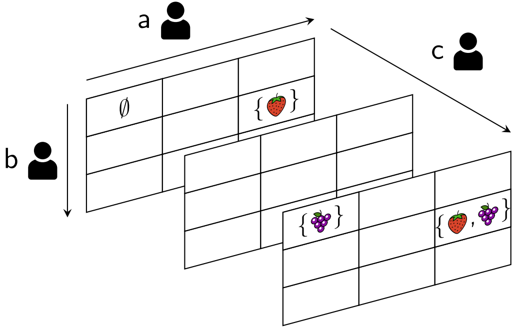

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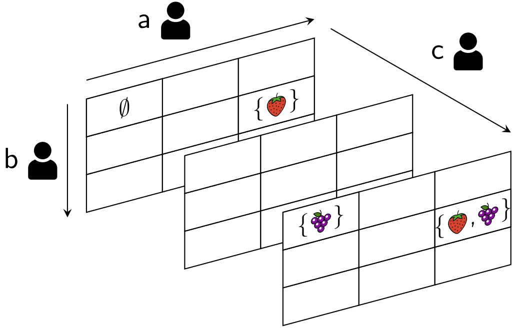

can be adapted for CDM, NL, DISAGREEMENT, PROMOTION

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- SFWork: survey of San Fransisco transportation choices
  Groups: live in city center, live in suburbs
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Model training

Optimize NLL using PyTorch's Adam with amsgrad fix
(Kingma & Ba, 2015; Reddi et al., 2018; Paszke et al., 2019)

Greedy algorithm fails in small examples

SFWork CDM Agreement

$C = \{\text{drive alone}, \text{transit}\}$

Greedy algorithm fails in small examples

SFWork CDM Agreement

$C = \{\text{drive alone}, \text{transit}\}$

Greedy

$Z = \{\text{carpool}\}$

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SFWork CDM Agreement

$$C = \{\text{drive alone}, \text{transit}\}$$

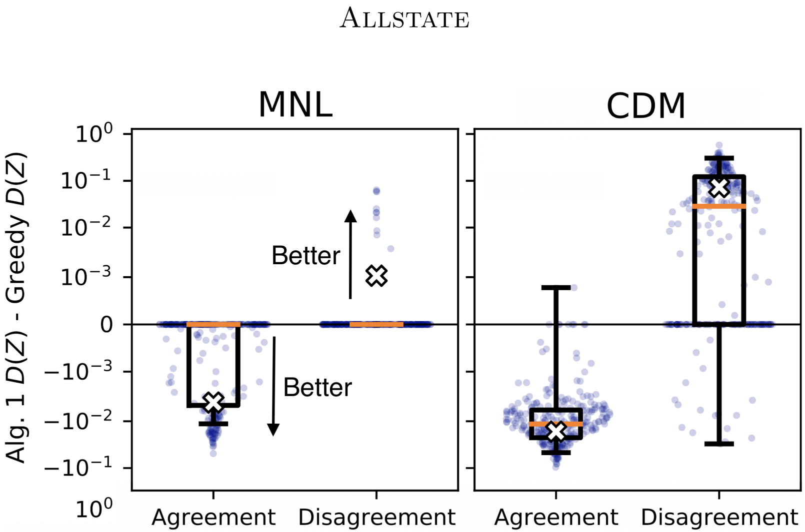
Greedy

$Z = \{\text{carpool}\}$

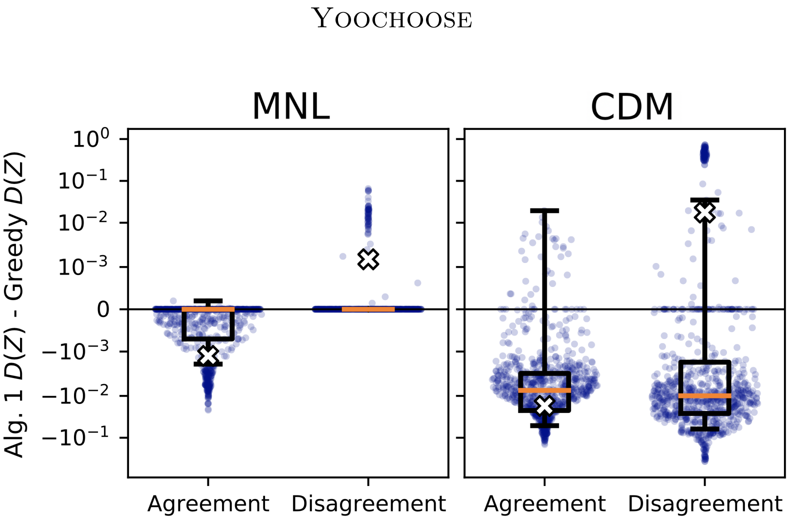
Optimal

$Z = \{\text{bike, walk}\}$

Approximation outperforms greedy on 2-item choice sets

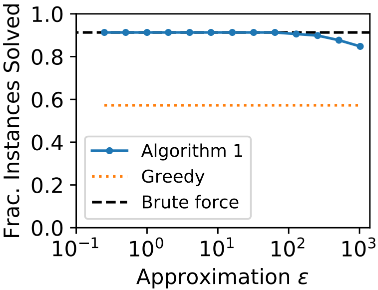


Approximation outperforms greedy on 2-item choice sets

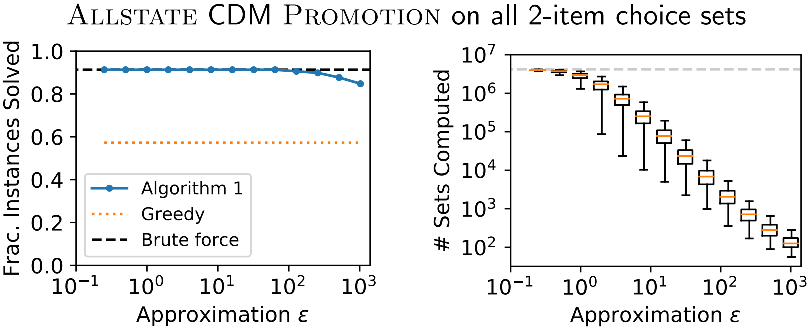


Approximation outperforms theoretical guarantee

Allstate CDM Promotion on all 2-item choice sets



Approximation outperforms theoretical guarantee



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Takeaways

1. Influence group preferences by modifying the choice set
2. NP-hard to maximize consensus or promote items
3. Promotion is easier than achieving consensus
4. Approximation algorithm that works well in practice

Availability

Data and source code hosted at `https://github.com/tomlinsonk/choice-set-opt`.