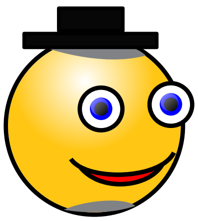

An argument, in limerick form, calculated to change the teaching of logic

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Introduction

In the '90s I did did engage
To lim'rick my way on this stage
At ZUM '95
I spoke at this dive
"A glimm'rick of hope" did I wage.

And now they have asked me once more
To amuse you a bit on this floor
Workshop do I teach?
Give keynoter speech?
[Naah]
More lighthearted fare was asked for

[Nevertheless]
I'll show you a thing —perhaps two
About the formality brew
A man of my age
Can be reckoned a sage
So advice do do I have for you

But first here's a test you must take
It's something to keep you awake
Please speak this math law
As a limerick saw
Subsequentally I will it spake

$$\frac{12 + 144 + 20 + 3\,\sqrt{4}}{7} + 5*11 = 9*9 + 0$$

The subject: teaching logic to beginners

I speak not about your research
That topic I leave in the lurch
I'm into another
—research's blood brother—
The teaching of logic's my perch

What interests me most, I must say,
Is pedagogogical play
How do we convey
To the kids of today
That logic indeed has cachet[1]

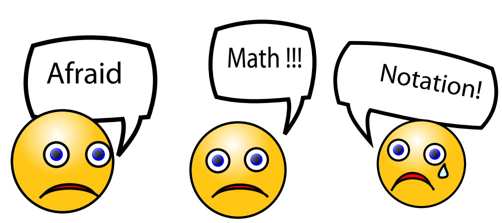
[Aah logic!]
{It's} a friend and a buddy and pal
What a wonderful boost to morale
To see mystery
Become absentee
When logic is used optimal

We argue that logic's the glue
that binds reas'ning methods into
a formidable foe
of confusion, and so ...
The world should be thinking this too

The current state of affairs

Alas as a general rule
We do not make logic seem cool
Discrete math is the place
Where logic has space
But in it we only outfool[2]

The students do enter the class
Afraid of notation and maths
At the end come out they
Feeling just the same way
And hating the logical paths



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[1] Cachet: high status, prestige, approval

[2] Outfool: v. t. To exceed in folly

"Why teach us this logic," they say?
"It's all an academic play
"It's not really used
"And we've been abused,"
They write on the course-end survey

What is our learning outcome?

The state of affairs does seem glum
We have to step back and think some
Perhaps we should ask
What is our real task?
What is our course learning outcome?

For logic the outcome should be
That students use logic with glee
A skill they've accrued
In making things proved
The beauty of logic they see

The logic we teach they will claim
Is useful in many domain
The students will feel
That logic's for real
And helps them develop their brain

The students will also acclaim
Developing proof's a neat game
It's opened their eye
[to] how math to apply
And now they know math's not arcane[3]

Logician's logics don't fit the bill

[But]The logics we're teaching today
Do not have the right propertay
Their use is not wide
In fact, I defied
You to use them in math ev'ry day

Some Natural Deduction Rules

$\wedge$-I: $\dfrac{P, Q}{P \wedge Q}$ $\wedge$-E: $\dfrac{P \wedge Q}{P}$ $\dfrac{P \wedge Q}{Q}$

$\Rightarrow$-I: $\dfrac{P1, \ldots, Pn \vdash Q}{P1 \wedge \ldots \wedge Pn \Rightarrow Q}$ $\Rightarrow$-E: $\dfrac{P, P \Rightarrow Q}{Q}$

E.g. Proof of: $p \wedge q \vdash p \wedge (q \vee r)$

1	p	$\wedge$-E, pr 1
2	q	$\wedge$-E, pr 2
3	$q \vee r$	$\vee$-I, 2
4	$p \wedge (q \vee r)$	$\wedge$-I, 1, 3

Logicians are not th'ones to blame
For they have a different aim
Logicians don't use
The logics they choose
To study the beasts is their game

"As point of departure you can
these logics review," said a man
This advice do we take
A thoughtful move make
And depart from them far as we can

Calculational logic

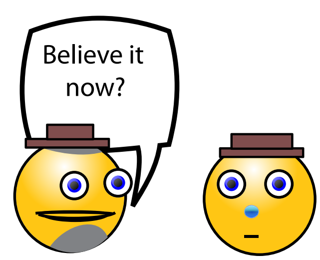
So what do the math people do
When they want to argue with you?
In their demonstration
They use calculation[4]
—[And] you're forced to agree with their view

Think (a+b) c not equal ac + bc? I'll show you!

(a+b) c
= <Symmetry, with b,a:= a+b,c>
c (a+b)
= <Left distributivity>
ca + cb
= <Symmetry, twice>
ac + bc

Your also asked why $\neg p \equiv p \equiv false$.

$\neg p \equiv p \equiv false$
= <(3.9), $\neg$ over $\equiv$, with q:= p>
$\neg(p \equiv p) \equiv false$
= < (3.3), Identity of $\equiv$ >
$\neg true \equiv false$ —(3.8), Def of false



[3] Arcane: requiring secret knowledge to be understood; mysterious; esoteric

[4] = is defined for all types. ≡ is used only for type boolean

Calculation's in many domain
Like sets and g'ometry plane
recurrence relations
[al]gebraic gyrations
—Ubiquitous is its nickname!

Inference rules of calculational logic

This calculululational form
Can be our lological norm
Its inference laws
And format can cause
Our logical thought to transform

Here is an inference rule
Of this calculational tool
Rule equal for equal
Is not so unus'al
It's used in the math in high school

Leibniz: $\dfrac{P \;=\; Q}{E[r{:=}\,P] \;=\; E[r{:=}\,Q]}$
(subst of equals for equals)

Another is trans(it)ivity
Of equals and equals, you see
Applications of it
Do say, I submit,
That last and first exps equal be

Transitivity of =:
$\dfrac{P = Q,\; Q = R}{P = R}$

A third rule's activitity
Gives the'rems from equality
If B equiv C
And the'rem is B
Then theorem also is C

Equanimity:
$\dfrac{B \equiv C,\;\; B}{C}$

Modus ponens:
$\dfrac{B \Rightarrow C,\;\; B}{C}$

That's better than modus ponens
Much nicer to use I contends
The rule will gain fame
(I gave it the name)
Equanimity's best of our friends

Of course modes ponens is there
As d'rived inference [rule], be aware
But now there's a pair
The stage it must share
With new Equanimity fair

So the logic we need does exist
Use it once and you cannot resist
To use it a lot
—The tool in your pot
With a nice calculational twist

But what about teaching the logic?

Teach slowly with passionate sway
One op is enough on each day
The students need drill
To acquire a skill
In developing proofs in this way

Equiv then negate —that's the way
Disjunction is on the third day
Conjunction is easy
Imply is so messy
It's last to be put into play

The axioms define the new op
New the'rems will then be brought up
The THING to discuss
With much detailed fuss
Is tactics for building proofs up

When all thru this stuff you have churned
Prop logic the students have learned
To you they'll direct
Their thanks and respect
Respect that indeed you have earned

Now show some techniques to apply
That do on the logic rely
Perhaps contradiction
[An]'tecedent assumption
"How wonderful!", students will cry

Proof techniques
1. Assume antecedent and prove consequent
2. Case analysis
3. Mutual implication
4. Contrapositive
5. Contradiction
6. Induction over natural numbers
7. Induction over a well-founded set

The predicate logic is next
Of course it is far more complex
But students have fun
And when it is done
You move on to other subjects

Show this is a good argument:

"Everybody loves my baby, but my baby loves nobody but me. So I am my own baby."
—Cliff Stoll, *Cuckoo's Egg*.

Define: p loves q: loves(p, q)
Cliff Stoll: S
his baby: B

Prove: $(\forall p|: loves(p, B)) \wedge (\forall p|: loves(B, p) \Rightarrow p = S) \Rightarrow B = S$

$(\forall p|: loves(p, B)) \wedge (\forall p|: loves(B, p) \Rightarrow p = S)$
$\Rightarrow$ ⟨Mono: Instantiation, with p:= B, twice⟩
$loves(B, B) \wedge (loves(B, B) \Rightarrow B = S)$
$\Rightarrow$ ⟨Modus ponens: $P \wedge (P \Rightarrow Q) \Rightarrow Q$⟩
$B = S$

Use same proof format in other areas

Each other discrete math domain
Has proofs that look roughly the same
And that is the thread
That all topics wed
That besews [bestows] on the course a nice frame[5]

A glimmerick of hope

The glimm'rick of hope is, for me,
That others look seriously
At math calculation
As logic foundation
And upgrade their pedagogy

I'm serious, this is the way
To teach students logic today
I beg you —attempt it
You will not regret it
Just try it and you'll say hooray!

I think that I better stop here
I'm sure that you have the idea
Of what I propose
To solve logic woes
I thank you for lending your ear

The test

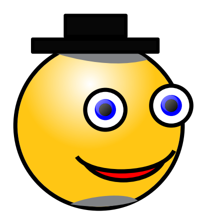
[Ah!]
Before I sit down I'll resolve
That puzzle I asked you to solve
How speak we this law
As a limerick saw?
Ingenuity it may involve

$$\frac{12 + 144 + 20 + 3\sqrt{4}}{7} + 5*11 = 9*9 + 0$$

A dozen, a gross, and a score
Plus 3 times the square root of 4
Divide it by 7
Add 5 times 11
gives 9 squared and not a bit more[6]

Questions?

I'm happy to answer your question
I'll do it of course, with discretion
But respect this event
and give your comment
As a well-spoken lim'rick expression




Note: The text "A Logical Approach to Discrete Math", by D. Gries and F.B. Schneider, Springer Verlag 1993, is good for teaching calculational logic and discrete math, as suggested in this speech. There, the logic is called equational logic.


[5] frame: the system around which something is built up

[6] This was a "puzzler" on the NPR show "Car Talk" in the United States in Spring 2011